Instructions: Try these problems from the textbook as practice (answers below).

Problems

3-8. Find the velocity, acceleration, and speed of a particle with the given position function. Sketch the path of the particle and draw the velocity and acceleration vectors for the specified value of t.

3. $\mathbf{r}(t) = \left\langle -\frac{1}{2}t^2, t \right\rangle$, $t = 2$
5. $\mathbf{r}(t) = 3\cos(t)\mathbf{i} + 2\sin(t)\mathbf{j}$, $t = \pi/3$
7. $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + 2\mathbf{k}$, $t = 1$

9-14. Find the velocity, acceleration, and speed of a particle with the given position function.

9. $\mathbf{r}(t) = \langle t^2 + t, t^2 - t, t^3 \rangle$
11. $\mathbf{r}(t) = \sqrt{2}t\mathbf{i} + e^t\mathbf{j} + e^{-t}\mathbf{k}$
13. $\mathbf{r}(t) = e^t(\cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k})$

15. Find the velocity and position vectors of a particle that has acceleration $\mathbf{a}(t) = \langle 2, 0, 2t \rangle$, initial velocity $\mathbf{v}(0) = 3\mathbf{i} - \mathbf{j}$, and initial position $\mathbf{r}(0) = \mathbf{j} + \mathbf{k}$.

Answers

3. $\mathbf{v}(t) = \langle -t, 1 \rangle$
   $\mathbf{a}(t) = \langle -1, 0 \rangle$
   $|\mathbf{v}(t)| = \sqrt{t^2 + 1}$.

5. $\mathbf{v}(t) = \langle -3\sin(t), 2\cos(t) \rangle$
   $\mathbf{a}(t) = \langle -3\cos(t), -2\sin(t) \rangle$
   $|\mathbf{v}(t)| = \sqrt{13}$.

7. $\mathbf{v}(t) = \langle 1, 2t, 0 \rangle$
   $\mathbf{a}(t) = \langle 0, 2, 0 \rangle$
   $|\mathbf{v}(t)| = \sqrt{4t^2 + 1}$.

9. $\mathbf{v}(t) = \left\langle 2t + 1, 2t - 1, 3t^2 \right\rangle$
   $\mathbf{a}(t) = \langle 2, 2, 6t \rangle$
   $|\mathbf{v}(t)| = |t|\sqrt{9t^2 + 8}$.

11. $\mathbf{v}(t) = \left\langle \sqrt{2}, e^t, -e^{-t} \right\rangle$
    $\mathbf{a}(t) = \left\langle 0, e^t, e^{-t} \right\rangle$
    $|\mathbf{v}(t)| = \sqrt{2 + e^{2t} + e^{-2t}}$.

13. $\mathbf{v}(t) = e^t \langle \cos(t) - \sin(t), \sin(t) + \cos(t), t + 1 \rangle$
    $\mathbf{a}(t) = e^t \langle -2\sin(t), 2\cos(t), t + 2 \rangle$
    $|\mathbf{v}(t)| = e^t\sqrt{(t+1)^2 + 2}$.

15. $\mathbf{v}(t) = \left\langle 2t + 3, -1, t^2 \right\rangle$
    $\mathbf{r}(t) = \left\langle t^2 + 3t + 1, -t, \frac{1}{3}t^3 + 1 \right\rangle$.